Real-time Optical Flow Estimation

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Abstract The gradient-based methods for optical flow estimation are based on a constraint equation which is defined for each image pixel. The structure of constraint equation make the problem ill-posed so in the past have been proposed some solutions based on regularization. On the contrary, under the assumption that in the immediate neighbourhood of a pixel the optical flow field is smooth, the constraint equations in that neighbourhood should have a common solution, in this case the problem is not ill-posed. Following this reasoning, an algorithm for evaluating the optical flow, which is suitable for parallel implementation is proposed in this paper. Moreover, parallel implementations of selected algorithms from the literature, for optical flow estimation, are presented in this paper with the intention of comparing their complexity and performance with respect to the proposed approach on a Connection Machine-2.

I. Introduction

The main problem of sequence analysis in vision is the estimation of the apparent motion usually called "velocity field" or "motion field" [1], which is the perspective projection of the 3D real velocity on the image plane. The estimation of the apparent velocity in a regular grid of the image is useful in solving many problems related to dynamic scene analysis such as 3D motion estimation and 3D object reconstruction, robot navigation, etc..

Recently, the necessity to perform motion analysis in real-time for robot navigation and other applications has provoked an interest in real-time estimation of the apparent motion. Parallel implementation is one way to achieve it. Presently, there are only few examples of parallel implementation for motion estimation in the literature, though, many computational approaches for motion estimation are highly parallelizable.

Three main approaches for solving the motion estimation problem, which are suitable for parallel implementation, can be identified in literature: matching (correspondence), spatio-temporal filtering, and the gradient-based.

In the first, local matching techniques are used to evaluate the displacements in subsequent frames for each element of the moving object (lines, corners, patterns) [2]. In [3] the matching approach has been implemented on a pyramidal processing architecture. The process starts at the coarsest level where the displacements are less than one pixel, passing from a level to a finer one by using a matching-based algorithm. Matching-based approach is also utilized in [4] as a first step for defining an algorithm for motion estimation on Connection Machine in close-to-real-time.

In the second approach, the estimation of motion is obtained by filtering both in the temporal and frequency domains. Filtering is tuned in frequency and space in order to detect the components of the motion. Typically, algorithms of this approach do not operate on local information, but use the global information contained in the image sequence [5], [6]. Though this approach is computationally complex, it fits well a pyramidal architecture, where a multi-resolution set of images from the scene under observation can be easily obtained [7], [8].

The third approach (i.e., gradient-based) provides a solution to motion estimation from the observation of brightness changes in the image plane, thus leading to motion estimation of image brightness features [9], [10], [11], [12]. The flow field of these features is normally called "optical flow". Generally, optical flow differs from the velocity field. On the other hand the estimation of an approximated velocity field such as an optical flow can be very useful in many cases. The gradient-base approach is suitable for parallel implementation since it requires access only to local image information. An example of is the fully pyramidal implementation presented by Enkelmann in [13]. As a first step, a pyramid of images of different resolutions is obtained by using a Gaussian filter. Then, starting from the lowest resolution images processing moves to higher resolutions, improving the optical flow estimation at each step. A parallel implementation on the Connection Machine-2 architecture (CM-2) of the algorithm presented by Tretiak and Pastor in [10] has been presented by Tistarelli in [14], providing quasi-real-time estimations. An example of hardware implementation can be found in Danielsson and al. [15], where a SIMD machine composed of 512 processors is proposed to estimate (512 × 512) optical flow fields in real-time.

The gradient-based approaches evaluate the optical flow by using the so called Optical Flow Constraint (OFC) equation:

$$E_x u + E_y v + E_t = 0, \qquad (1)$$

where the abbreviation for partial derivatives of the image brightness has been introduced, and u, v correspond to dx/dt, dy/dt, and represent the components of the local velocity vector $\mathbf{V}$ along the x and y directions, respectively. The definition of the OFC is derived from the observation that the changes in the image brightness $E(x(t), y(t), t)$ are supposed to be stationary with respect to the time variable (i.e., $dE/dt = 0$). In general, boundary and smoothness constraints are needed to obtain a computational solution for the OFC.

Optical flow estimation is susceptible to two main difficulties. The first involves the discontinuities in the local velocity, relating to image brightness discontinuities, which are originated by the presence of noise, too crisp patterns on the moving object surfaces, occlusions between moving objects. Generally, this difficulty can be overcome (or attenuated) by convolving the image with a 2-D or 3-D Gaussian smoothing operator. The second difficulty is the so-called "problem of aperture", which is also present in the human vision [9].

In this paper, a new algorithm for optical flow estimation in real-time and its parallel implementation is proposed (Sec. IV). In addition, new parallel implementations of two well-known algorithms for optical flow estimation, selected from the literature, are presented (Sec. II and III). All the three algorithms are compared on a SIMD architecture such as the CM-2 (Sec. V). The major features of these three algorithms are summarised as follows:

(a) the approach of Horn and Schunck [9], is a *regularization-based* algorithm, where the optical flow estimation problem, by using the OFC (i.e., $dE/dt = 0$);

(b) the approach of Tretiak and Pastor [10], is a *multiconstraint-based* algorithm, where the optical flow is obtained by solving a determined system of constraint equations (i.e., $d\nabla E/dt = 0$) at each image pixel;

(c) the proposed approach is a *multipoint-based* algorithms. This approach is derived from the multipoint technique for solving partial differential equation.

The reference machine used for the parallel implementations of these algorithms is the CM-2, a SIMD machine with a processing element (PE) for each image pixel, and an efficient communication among processing elements. On such machine all the calculations involved in the algorithms described are performed simultaneously by the PEs assigned to the image pixels.

II. Regularization-Based Algorithm

The *regularization-based* approaches consider the optical flow estimation an ill-posed problem [9], [16], [17]. Solutions are obtained minimizing a functional, where a smoothness constraint is appropriately weighted to regularize the solution. Usually, these methods lead to iterative solutions, and the velocity is evaluated in every point of the image. The drawbacks of these approaches are related to the fact that difficulties occur in the presence of object occlusions, and the depth of propagation of the field depends on the number of iterations and on the weighting factor. Horn and Schunck solution [9] is taken as representative of this class of algorithms. The defined functional in this case is:

$$F = \int\int \left[(E_x u + E_y v + E_t)^2 + \alpha^2(u_x^2 + u_y^2 + v_x^2 + v_y^2)\right] dxdy, \tag{2}$$

where the first term is the OFC, the second term is taken as a measure of the goodness of OFC approximations, and α is a weight factor to control the influence of the smoothness constraint. This functional was minimized by using calculus of variations. That approach leads to a system of two coupled differential equations from the Euler Lagrange equations. These equations can be decoupled and solved iteratively by using the discrete approximation of the Laplacian, whereby a couple of iterative equations is obtained by using a finite difference method. These are then used to estimate the optical flow components:

$$u^{n+1}_{i,j,t} = \bar{u}^n_{i,j,t} - \frac{E_{xi,j,t}\left[E_{xi,j,t}\bar{u}^n_{i,j,t} + E_{y_{i,j,t}}\bar{v}^n_{i,j,t} + E_{ti,j,t}\right]}{\left(\alpha^2 + E^2_{xi,j,t} + E^2_{y_{i,j,t}}\right)},$$

$$v^{n+1}_{i,j,t} = \bar{v}^n_{i,j,t} - \frac{E_{y_{i,j,t}}\left[E_{xi,j,t}\bar{u}^n_{i,j,t} + E_{y_{i,j,t}}\bar{v}^n_{i,j,t} + E_{ti,j,t}\right]}{\left(\alpha^2 + E^2_{xi,j,t} + E^2_{y_{i,j,t}}\right)},$$

where: $E_{xi,j,t}$, $E_{y_{i,j,t}}$ and $E_{ti,j,t}$ are estimated by using equations:

$$\begin{aligned} E_{xi,j,t} &= (E_{i+1,j,t} - E_{i-1,j,t})/2, \\ E_{y_{i,j,t}} &= (E_{i,j+1,t} - E_{i,j-1,t})/2, \\ E_{ti,j,t} &= (E_{i,j,t+1} - E_{i,j,t-1})/2, \end{aligned} \tag{4}$$

where, only the estimations of $E_{xi,j,t}$, and $E_{y_{i,j,t}}$ require the communication of the pixel data from neighbouring PEs; and:

$$\begin{aligned} \bar{u}^n_{i,j,t} = {} & (u_{i-1,j-1,t} + u_{i-1,j+1,t} + u_{i+1,j-1,t} + u_{i+1,j+1,t})/12 + \\ & (u_{i-1,j,t} + u_{i+1,j,t} + u_{i,j-1,t} + u_{i,j+1,t})/6, \\ \bar{v}^n_{i,j,t} = {} & (v_{i-1,j-1,t} + v_{i-1,j+1,t} + v_{i+1,j-1,t} + v_{i+1,j+1,t})/12 + \\ & (v_{i-1,j,t} + v_{i+1,j,t} + v_{i,j-1,t} + v_{i,j+1,t})/6, \end{aligned}$$

where n is the iteration number. In this iterative solution a guessed value for optical flow estimation at time t can be obtained from the previous time-step (i.e., $\bar{u}^0_{i,j,t} = u^{\omega}_{i,j,t-1}$, where ω is the number of iterations executed at the previous time step).

Complexity

The parallel solution of Horn and Schunck algorithm is composed of two phases. In the first estimations of the image brightness derivatives are calculated. The second is an iterative calculation process defined by equations (3). It should be noted that both these phases involve communication of data among neighbouring PEs.

The explicit complexity of this solution is strongly dependent on the number of the iterations, I_t, and results on a sequential machine in $C() = 3M^2 + I_t M^2$, where M is the image dimension; the first term is due to the estimation of the partial derivatives for the image brightness, and the second to the iterative process for evaluating equations (3). On a mesh-connected parallel architecture like the CM-2 with $M \times M$ processing elements the asymptotical complexity is $O(I_t)$. The number of floating point operations (FLOP) which have to be executed at each iteration by each PE is only 49.

III. Multiconstraint-Based Algorithm

The ***multiconstraint-based*** approaches for optical flow estimation are based on the observation that the condition $dF/dt = 0$ can be made valid for any motion-invariant function F such as contrast, entropy, average, variance, etc., instead of the image brightness, E, in the OFC. By using a set of these constraints, which are evaluated at the same point in the image, an over-determined set of equations with u and v as unknowns can be obtained [18]. Other methods derive constraint equations which can be regarded as obtained by taking the first-order derivatives of OFC with respect to x, y and t [10], [11], [12]. These multiconstraint-based approaches use traditional numerical methods for the inversion or pseudo-inversion of the coefficient matrix of the set of equations. In general, most of the multiconstraint-based algorithms are suitable for parallel implementation on SIMD architectures, since they use only local information.

The solution of Tretiak and Pastor [10] adopts a couple of constraint equations which can be obtained by taking the derivative of the OFC equation with respect to x and y, and neglecting the first-order derivatives (u_x, u_v, v_x, v_y) of the velocity components [15] or through a different path by using the equations of motion components such as in [11]:

$$\begin{aligned} E_{xx}u + E_{xy}v + E_{xt} &= 0, \\ E_{xy}u + E_{yy}v + E_{yt} &= 0. \end{aligned} \qquad (5)$$

The solution is obtained directly through the inversion of the matrix of coefficients H (i.e., the Hessian). Thus, only for those points where the determinant of H ($g_c = E_{xx}E_{yy} - {E_{xy}}^2$) is different from zero a solution of the optical flow is provided. The components of the optical flow field are estimated is each pixel by using:

$$\begin{aligned} u_{i,j,t} &= \frac{E_{xy_{i,j,t}}E_{yt_{i,j,t}} - E_{yy_{i,j,t}}E_{txi,j,t}}{g_{c_{i,j,t}}}, \\ v_{i,j,t} &= \frac{E_{xy_{i,j,t}}E_{txi,j,t} - E_{xxi,j,t}E_{yt_{i,j,t}}}{g_{c_{i,j,t}}}, \end{aligned} \qquad (6)$$

where the second-order partial derivatives of the image brightness are estimate by using the central difference on the first-order derivatives – e.g., $E_{xy_{i,j,t}} = (E_{xi,j+1,t} - E_{xi,j-1,t})/2$. This solution is very sensitive to discontinuities, since it uses the second-order derivatives of the image brightness. To improve performance, an image convolution with a Gaussian filtering was adopted in [10], [15], [14], and also in our experiments.

Complexity

The parallel solution to this algorithm is composed of three phases: the first is the convolution of the images with a Gaussian filter, the second is the estimation of the first- and second-order image brightness derivatives, and the third is the estimation of the velocity components from equations (6). The first two phases involve data communication among neighbouring PEs.

The explicit complexity of this solution is strongly dependent on the dimension of the Gaussian pattern which is usually convolved with the image, prior to the optical flow estimation. At any time instant only one new im: is filtered, while the optical flow estimation is calcula for the previous sequence. The explicit complexity volved on a sequential machine is $C() = 2F^2M^2 + 3M$ $5M^2 + M^2$, where M is the image dimension, and *I* the dimension of the Gaussian filtering pattern. The f term of the above equation is due to the Gaussian filteri the second to the estimation of the first-order derivat of the image brightness, the third to the estimatioı the second-order derivatives of the image brightness (*I* E_{yy}, E_{xy}, E_{tx}, E_{ty}), and the forth to the estimatio the optical flow by using equations (6). The asympt cal complexity on a parallel architecture such as a n of $M \times M$ PEs is $O(F^2)$. The number of floating p operations that must be executed by each PE to estir the optical flow and filter the results are $2F^2 + 27$. As be seen in Table 1, increasing the dimension of the f mask, the filtering stage becomes the dominant (dı the intensive communication required among PEs w the filtering mask boundaries), compared to the cal tion effort in image brightness derivatives and optical estimation which takes only about 22 msec. on a (with 64 floating point unit and a vp-ratio of 1:4.

F	5	7	9	11	13
Time in msec.	22	60	108	183	291

Table 1: Timing expressed in milliseconds for the con lution of the ~~i~~mage with a pattern with dimension $F \times$ on CM-2 with vp-ratio 1:4.

IV. Multipoint-Based Algorithm

Considering that the optical flow changes follow which is approximately linear, a smoothed solution fc tical flow estimation can be obtained from a linear ap imation of the adopted constraint in the neighborhc the point under consideration. This assumption is only if the optical flow field under observation is sn Then, the constraints evaluated in a set of neighl pixels at a certain point represent the same veloc a first approximation. This approach was called point, and in literature several cases are presente [20], [12].

In this section, an improved version of the mult based algorithm proposed by Del Bimbo, Nesi, an in [21] has been presented and parallelized. Consid generic point $\mathbf{p}$ on the image plane having velocit ponents (u, v). If the optical flow changes followin which is approximately linear in x, y, then each p the neighbourhood of $\mathbf{p}$ has approximatively the sa locity components of $\mathbf{p}$ [21], hence, an over-dete system of $N \times N$ OFC equations:

$$E_{t(i,j,t)} + E_{x(i,j,t)}u + E_{y_{(i,j,t)}}v = 0,$$

can be defined, where N is the dimension of age neighbourhood around the point $\mathbf{p}$ (if N is $i = -(N-1)/2, .., (N-1)/2, j = -(N-1)/2, .., ($). This over-determined system of $N \times N$ equati 2 unknowns ($N \geq 2$), which are the velocity com

(u, v) of $\mathbf{p}$. The solution of this over-determined system of equations is obtained by means of a least-squares technique in each estimation point.

Augmenting the neighbourhood dimension, N, around the pixel under consideration smoother optical flow fields can be obtained. On the other hand, large N values lead to a loss in resolution on the moving object boundaries.

The presented algorithm is less sensitive to the discontinuities than the methods which use the second-order partial derivative of the image brightness with the same neighbourhood dimension, N [21]. On the other hand, inaccurate results can be obtained, since the estimation of the optical flow field is computed in a pixel neighbourhood, disregarding the possible difference in velocities. Therefore, it can be used safely only when the optical flow is smooth.

Parallel implementation

The multipoint-based algorithms work locally on the immediate neighbourhood of each pixel, and thus can be profitably mapped on a mesh architecture, where a PE is assigned to each pixel of the image. Corresponding pixels of three consecutive images which belong to the same time window used (to estimate the partial derivatives of the image brightness in (4)) are stored in each PE. Thus each PE can directly manages the time history of the corresponding to its pixel (4c).

After the estimation of the image brightness derivatives, each PE (pixel) has an OFC equation. Then, each PE receives from the $N \times N$ neighbouring PEs the coefficients of their OFC equations. Every PE has in this way an over-determined set of $N \times N$ OFC equations in 2 unknowns:

$$AV + K = 0.$$

where $\mathbf{V}$ is the optical flow vector with components u, v; $A \in \mathcal{R}_{N^2 \times 2}$ matrix of coefficients, with $a_{r,1} = E_{x_r}$ and $a_{r,2} = E_{y_r}$; and $K \in \mathcal{R}_{N^2}$ vector with known terms $k_r = E_{t_r}$ for $r = 1, .., N^2$. An increase in N, the neighbourhood size considered, leads to a significant increase in memory requirements to store the matrix and vector elements at each PE (for instance, with $N = 7$, 147 memory locations are required for storing the coefficients of matrix A and vector K above).

The solution of the over-determined system of equations by using the least-squares technique consists of minimizing the norm:

$$\|A\mathbf{V} + K\|^2.$$

This is performed by using the pseudo-inverse technique transforming the above system of equations into:

$$A\mathbf{V} + K = 0, \tag{7}$$

where $A = A^T A$, and $K = A^T K$ (i.e., A^T is the transpose of A). This system of equations can be solved by using traditional techniques such as LU decomposition, Gauss Jordan, etc.. In our case the system (7) is composed of 2 equations in 2 unknowns, and the direct solution was adopted. In particular, the coefficients of the matrix A and of the vector K are estimated by using:

N	3	5	7	9
Time in msec.	45	122	268	469

Table 2: Execution time expressed in milliseconds and number of operations for the multipoint algorithm depending on the dimension of the neighborhood area dimension N, with vp ratio 1:4.

$$\hat{a}_{i,j} = \sum_{r=1}^{N^2} a_{i,r}^T a_{r,j} = \sum_{r=1}^{N^2} a_{r,i} a_{r,j},$$

$$\hat{k}_i = \sum_{r=1}^{N^2} a_{i,r}^T k_r = \sum_{r=1}^{N^2} a_{r,i} k_r.$$

The estimation of the $\hat{a}_{i,j}$ and $\hat{k}_i$ (for $i = 1, 2$; $j = 1, 2$) can be performed by accumulating one term at a time, from the r-th neighbouring OFC equation (for $r = 1, .., N^2$), to obtain the final sum, thereby avoiding the need to store the entire set of N^2 OFC coefficients at each PE's memory.

In the process of accumulation an OFC which has an E_t less than a chosen threshold is ignored as an insignificant constraint equation. Also the constraints which have too large values for E_x and E_y are neglected.

Complexity

The explicit complexity for the presented multipoint solution on a sequential machine to estimate a velocity vector for each pixel in an $M \times M$ image is $C() = 3M^2 + 3M^2N^2 + 8M^2$, where the first term corresponds to the estimation of the partial derivatives of the image brightness; the second term is due to the least-squares technique for calculating $\hat{a}_{i,j}$ and $\hat{k}_i$ (for $i = 1, 2$, and $j = 1, 2$); and the third is due to the method for solving the final system of equations (7). The asymptotical complexity of the multipoint solution on a sequential machine is of $O(M^2N^2)$.

On a parallel architecture, such as the CM-2, with one PE per image pixel, the asymptotical complexity is reduced to $O(N^2)$, obtaining a respective Speed-Up of about M^2. Table 2 presents the dependence of the execution time of the algorithm on N, the neighbourhood size considered. The number of floating point operations required does not depend on the dimension of the image. For $N = 5$ about 270 FLOP (floating point operations) are made by each PE in estimating the optical flow value. To estimate the optical flow at video rate (25 times per second) the calculations should be completed within $40msec.$, demanding a capability of 6750 FLOPS from each PE for a real-time implementation.

V. Experimental Comparisons

In this section a performance evaluation is provided for the three algorithms discussed in previous sections. Qualitative comparison of the results obtained by the three

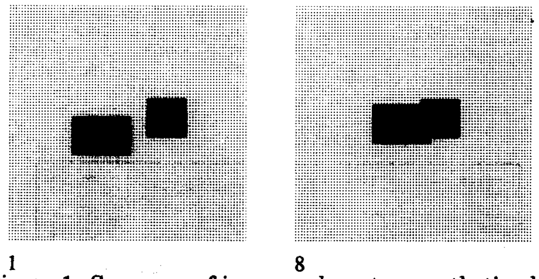

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Figure 1: Sequence of images where two synthetic objects with superimposed plaid pattern move in opposite directions, (180 and 45 degrees with respect to the X-axis, respectively), (1st and 8th frame, with 128 x 128 image resolution).

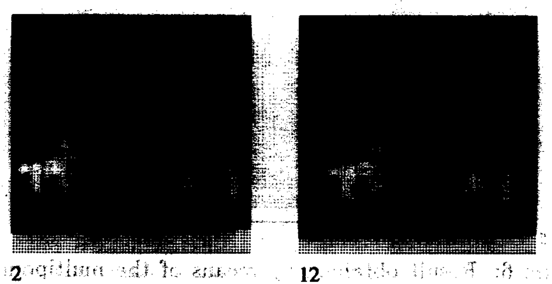

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Figure 2: Sequence of images where real objects are moving in different directions (2nd, and 12th frame, with 128 x 128 image resolution).

algorithms when applied to the test images is offered for selected test cases, along with a comparison of their complexity and the efficiency of their parallel implementation on the CM-2.

The first test sequence is that of two synthetic objects with a superimposed plaid pattern, which are moving in opposite directions. The plaid pattern consists in the combination of two sinusoidal patterns with orthogonal directions. This sequence was designed to test the performance in the case of occlusion (see Fig.1). The second test sequence (see Fig.2), presents two moving objects which are moving of translational motion in opposite directions (e.g., the toy dog and the little parallelepipedos). This test sequence has been chosen to test the performance in the case of noisy images.

The *multiconstraint-based* solution of Tretiak and Pas-or [10], is highly sensitive to noise, using the second-order lerivatives of the image brightness. This is clearly demontrated in Fig.3. The optical flow estimation obtained ith this algorithm is inaccurate at the objects' boundries, particularly at the boundary between the occluding bjects.

The *regularization-based* solution of Horn and Schunck], may produce a smooth optical flow estimation in the esence of noise by increasing the number of iterations, the α value (see Fig. 4).

However, doing so has a negative effect at objects undaries, which are obtained diffused and with erroneous optical flow estimation (see Fig.5).

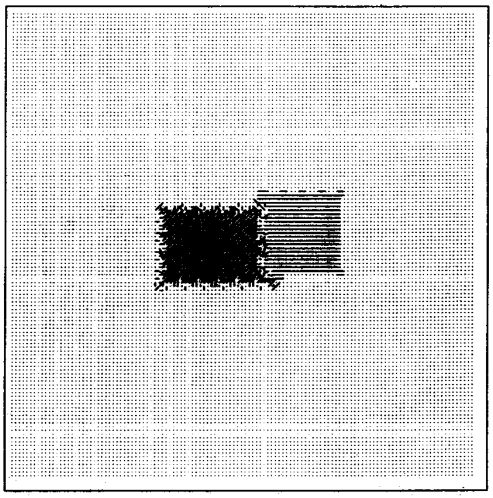

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Figure 3: Result obtained by means of the Tretiak and Pastor algorithm (i.e., multiconstraint-based) with respect to the first test case (7th frame), ($F = 9$).

For the test case of Fig.2, the *multipoint-based* technique produces an accurate segmentation of the moving objects (sharp boundaries) in image areas where signal to noise ratio is high (see Fig.6).

A smoother solution can be obtained with the multipoint algorithm by augmenting the dimension of the neighborhood around each pixel used in the estimation. However, this is also the cause of a loss in resolution at the objects boundaries, and an increase in the computational effort. This technique is less sensitive to discontinuities with respect to the multiconstraint-based approach (see Fig.6).

In the presence of occlusion, at the border between two moving objects, the two objects contribute conflicting velocities, and taking the least-squares estimation yields an inaccurate optical flow estimation, deviating from both (see Fig.7).

Table 3 provides a comparison of the algorithms in terms of complexity and efficiency of implementation. For the multipoint-based algorithm real-time performance is obtained, and the complexity is of N^2, with N the dimension of image segment used in the optical flow estimation. The optical flow estimation with Tretiak and Pastor algorithm is obtained in real-time, but overall efficiency is degraded by the need to filter the images with a large Gaussian filter.

By using the algorithm of Tretiak and Pastor, it is needed to use large values of F to produce qualitatively comparable results with respect to those obtainable by means of the proposed multipoint algorithm ($N = 5$, $F = 9$). The complexity of Horn and Schunck algorithm is proportional to the number of iterations required to achieve a stable estimation, which is normally of the order of 100 (larger than N^2).

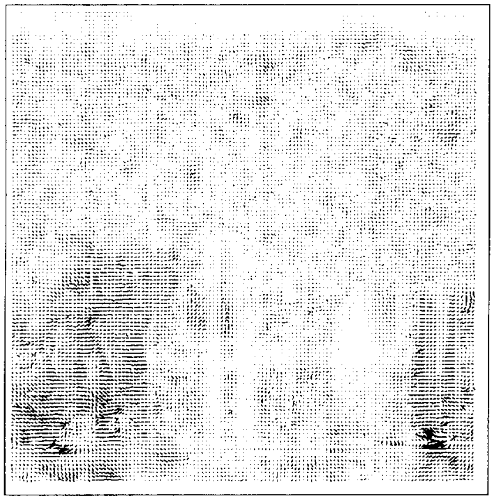

Figure 4: Result obtained by means of the Horn and Schunck algorithm with respect to the second test case (50 iterations, $\alpha = 2.0$), (7th frame).

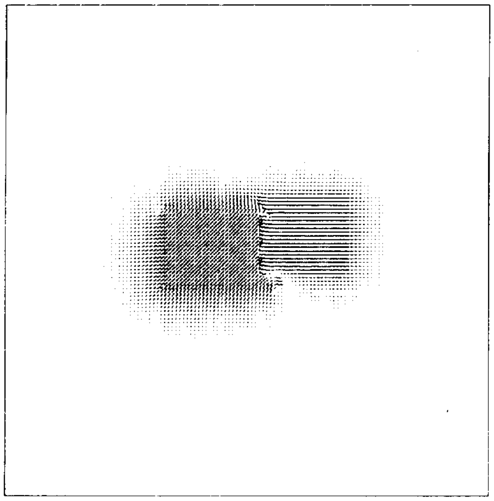

Figure 5: Result obtained by means of the Horn and Schunck algorithm with respect to the first test case (100 iterations, $\alpha = 2.0$), (5th frame).

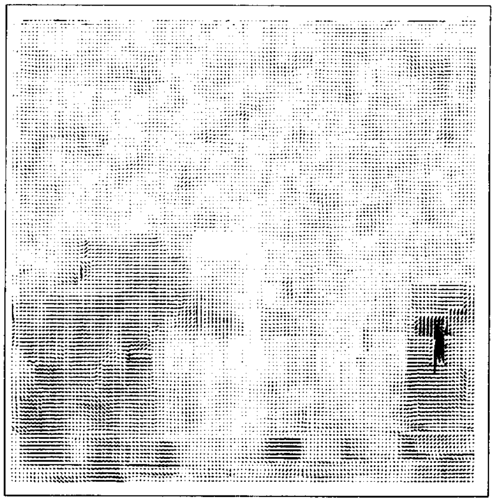

Figure 6: Result obtained by means of the multipoint-based algorithm with respect to the second test sequence with $N = 7$ (5th frame).

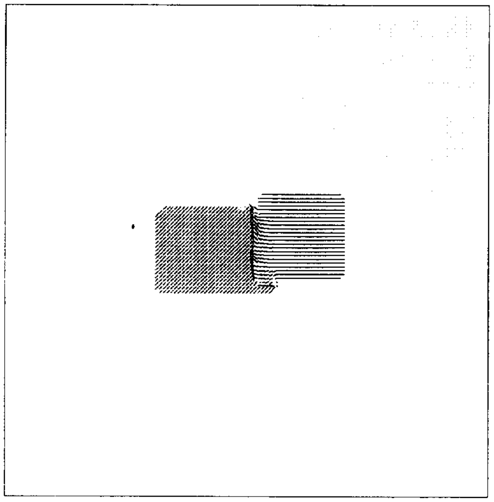

Figure 7: Result obtained by means of the multipoint-based algorithm with respect to the first test sequence with $N = 5$ (7th frame).

	O ()	E_Time	Time	FLOP/PE	MFLOPS
Mult.	N^2	120	30	136	19.4
T & P	F^2	130	32.5	108	13.6
H & S	I_t	$15I_t$	$3.75I_t$	$6+49I_t$	53.5

Table 3: A comparison of the presented algorithms, in terms of complexity and execution time: E_Time – elapsed time (in millisecond) on the CM-2 with vp-ratio 1:4 (i.e., 4096 PE); Time elapsed PE time that could be obtained having one PE per pixel. Parameters: the multipoint-based algorithm $N = 5$ and $G = 3$; Tretiak and Pastor algorithm $F = 9$.

Referring to the other entries of Table 3, the best performance in terms of number of computations involved (floating point operations - FLOP) is obtained by Tretiak and Pastor algorithm. The Horn and Schunck algorithm requires few FLOPs/PE per iteration, but generally the number of iterations required is large (e.g., $I_t = 100$). The best rate of computations (MFLOPS) is obtained by Horn and Schunck algorithm.

With images produced at a video rate (i.e., 25 frames per second, or 0.04 sec./frame), real-time performance entails the calculation of optical flow estimation each $40msec$. On the CM-2 this target is met by the multipoint-based with least-squares estimation algorithm, and by the Tretiak and Pastor algorithm (in the latter, when only the optical flow estimation and a light filtering are considered).

VI. Conclusions

A multipoint-based approach for optical flow estimation has been presented together with its parallel implementation. Moreover, parallel implementation of two representative algorithms of the gradient-based approach for optical flow estimation have been presented. All three parallel implementations have been profitably implemented on the SIMD architecture of the Connection Machine-2. Real-time performance has been obtained for the proposed multipoint-based algorithm and for the multiconstraint-based algorithm of Tretiak and Pastor.
In terms of qualitative results, it has been found that the multipoint-based is less susceptible to object occlusion and noise than the other algorithms discussed, while the multiconstraint-based algorithm of Tretiak and Pastor has been found the most susceptible to these difficulties.

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